Program:	**F.Y.B.Sc IT**	**Semester:**	**II**	**Program Code:**	**UGIT01**
Course:	**Linear Algebraic Structures and Predictive Modelling**			**Course Code:**	**NUIT204**

Duration: 1 Hour **Examination Pattern:** NEP-Autonomous-External **Max. Marks:** 30

Examination:

REGULAR

Instructions:

1. All questions are compulsory with internal choice.
2. All symbols and notations used, adhere to standard conventions.
3. Scientific calculator fx 82 series or lower version is only permitted.
4. Statistical table will be provided on request.

Q. 1		Attempt any THREE of the following.	[15]	Course Outcome	Knowledge Level
	(a)	A robotics lab is studying the motion of a robotic arm in 3D space. The position vectors of the arm's endpoint are recorded as points in $\mathbb{R}^3$. Due to a mechanical constraint, the arm can only reach positions $p = [x, y, z]^T$ that satisfy the linear relationship: $2x + y - z = 0$. Let S be the set of all position vectors in $\mathbb{R}^3$ that satisfy this constraint. (i) Prove that S is a subspace of $\mathbb{R}^3$ by verifying all three subspace conditions. (ii) Find two linearly independent vectors that span S, and verify they form a basis for S. (iii) The robotic arm is currently at position $[3, -2, 4]^T$. Determine whether this position is reachable under the given constraint. Explain mathematically.		CO1	L1
	(b)	A population ecology study tracks two competing species: rabbits (R) and foxes (F) in a controlled ecosystem. The population dynamics from one month to the next are modelled by the linear transformation represented by the matrix: $$A = \begin{bmatrix} 1.2 & -0.2 \\ 0.3 & 0.7 \end{bmatrix}$$ where the population vector $v = [r, f]^T$ represents the number of rabbits and foxes (in hundreds), and the next month's population is given by $v' = Av$. (i) Find the characteristic equation of matrix A. (ii) Calculate both eigenvalues of A. (iii) For each eigenvalue found, determine the corresponding eigenvector. (iv) Interpret the biological significance: If the initial population is an eigenvector corresponding to the larger eigenvalue, what does this tell us about the long-term behaviour of the ecosystem? Explain in terms of population growth or decline.		CO2	L2
	(c)	If $v_1 = (1, 2, 3)$ and $v_2 = (4, 5, 6)$, check if these vectors are linearly independent.		CO1	L1
	(d)	Let $T: \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = (2x + 3y, x + 3y)$. Find the matrix representation of T.		CO1	L1

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(e) A tiny 2×2 image patch has grayscale intensity values $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$. Each entry represents the brightness of a pixel (0 = black, larger values = brighter). — CO2, L2

(i) Compute the singular values of A.

(ii) Compute the dominant left singular vector u_1.

(iii) Write the rank-1 SVD approximation $A_1 = \sigma_1 u_1 v_1^T$.

(iv) Interpret what the dominant singular value says about the main pattern in this image patch. (e.g. whether the image is mostly uniform or if a particular pattern dominates).

Q. 2		Attempt any THREE of the following.	[15]	Course Outcome	Knowledge Level

(a) A machine learning engineer is developing a predictive model for battery life in autonomous drones. She collects data on 8 drone flights and records two variables: — CO3, CO4, L3

- Flight Time (x): Duration of flight in minutes
- Battery Temperature (y): Average battery temperature in °C during flight

The data collected is shown below:

Flight	1	2	3	4	5	6	7	8
Flight Time (x)	15	20	25	30	35	40	45	50
Battery Temp (y)	35	38	42	45	48	52	55	58

Given the following computed values:

$\Sigma x = 260, \Sigma y = 373, \Sigma x^2 = 9500, \Sigma y^2 = 17855,$

$\Sigma xy = 12820, n = 8$

(i) Calculate the Pearson correlation coefficient (r) between Flight Time and Battery Temperature.

(ii) Interpret the correlation coefficient value obtained. What does it indicate about the relationship between flight time and battery temperature?

(iii) Using the least squares method, find the simple linear regression equation $\hat{y} = a + bx$.

(b) A cybersecurity team is developing an AI-based email spam detection system. They train a logistic regression model to classify emails as spam (y = 1) or legitimate (y = 0) based on the number of suspicious keywords found in the email. — CO4, L5

The trained logistic regression model has the following equation:

$log \frac{p}{1-p} = -2.5 + 0.8x$

where:

- p = probability that an email is spam
- x = number of suspicious keywords in the email
- log denotes the natural logarithm (ln)

(i) Write down the logistic regression equation in the form $p = \frac{1}{1+e^{-z}}$. What is the expression for z in this model?

(ii) Calculate the probability that an email is spam when it contains:

- $x = 2$ suspicious keywords
- $x = 5$ suspicious keywords

(iii) Calculate and interpret the odds ratio when the number of suspicious keywords increases from 2 to 3.

(iv) The cybersecurity team sets a classification threshold at $p = 0.5$. Determine the minimum number of suspicious keywords required for an email to be classified as spam. Show your calculation.

(c) If $X \sim N(\mu = 25, \sigma = 10)$, find — **CO4** — **L5**

(i) $P(X \geq 15)$ (ii) $P(25 < X < 35)$ (iii) $P(15 < X < 35)$

(d) A senior citizen receives on average 2.5 telephone calls during his afternoon nap period 1400-1405 hrs. Find probability that on a certain day, he receives — **CO4** — **L5**

(i) no telephone calls.

(ii) exactly four calls during the same period.

(e) An autonomous robot navigation system uses a Naive Bayes classifier to predict whether a detected object is an Obstacle or Safe Path based on sensor readings from three different sensors. — **CO6** — **L6**

Class Variable: $C \in \{Obstacle, Safe\ Path\}$

Feature	Description	Possible Values
F_1	Infrared sensor reading	High, Low
F_2	Ultrasonic distance	Near, Far
F_3	Camera edge detection	Detected, Not Detected

Given Probabilities:

Class Priors: P(Obstacle) = 0.4 P(Safe Path) = 0.6

Conditional Probabilities (Likelihoods):

Feature	Value	P(Value|Obstacle)	P(Value|Safe Path)
F_1	High	0.85	0.15
F_1	Low	0.15	0.85
F_2	Near	0.90	0.20
F_2	Far	0.10	0.80
F_3	Detected	0.75	0.30
F_3	Not detected	0.25	0.70

The robot's sensors report the following readings:

- Infrared sensor: High
- Ultrasonic distance: Near
- Camera edge detection: Detected

(i) Calculate the unnormalized posterior probability for the class "Obstacle" given the sensor readings.

(ii) Calculate the unnormalized posterior probability for the class "Safe Path" given the sensor readings.

(iii) Normalize both probabilities and determine which class the robot should predict. Calculate the final posterior probabilities.

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