FYIT/SEM II/EXT/NM

Time: 2½ Hrs. **Marks:75**

Note: (1) All questions are compulsory with internal choice.
(2) Symbols have their usual meanings.
(3) Scientific calculator fx 82 series or lower version is only permitted.

Q.1 Attempt Any Three of the following. (15)

(a) Define accuracy and precision. What are round-off errors? Explain.

(b) Use zero through third order Taylor series expansions to approximate the function $f(x) = 25x^3 - 6x^2 + 7x - 88$ from $x_i = 1$ with $h = 2$. That is, predict the function's value at $x_{i+1} = 3$.

(c) Explain conservation laws and engineering problems.

(d) Evaluate and interpret the condition number for $f(x) = \frac{\sin x}{1+\cos x}$ for $x = 1.0001\pi$

(e) Explain the terms:
(i) Significant figures (ii) Formulation errors (iii) Data uncertainty
(iv) Truncation error (v) Absolute error

(f) Evaluate $y = x^3 - 7x^2 + 8x - 0.35$ at 1.37 use 3 digit and 4 digit arithmetic and find the significant digits lost. Also, find the relative error for each approximation.

Q.2 Attempt Any Three of the following. (15)

(a) Find the roots of the equation $x^3 - 12.2x^2 + 7.45x + 42 = 0$ between 11 and 12 using Regula-Falsi method. (Five iterations).

(b) Obtain the root of the equation $2x^3 - 5x^2 + 5x + 3 = 0$ using Newton–Raphson method. (Five iterations). Take initial root as $x_0 = 0$.

(c) Determine the real root of $4x^3 - 6x^2 + 7x - 2.3 = 0$ using bisection method correct upto three decimal places.

(d) The following table shows the number of students and range of marks. Find the number of students who have secured less than 45 marks.

Marks	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
No. of students	31	45	32	27	15

(e) For the following data, find $f(7.5)$ using Newton's backward interpolation formula.

x	1	2	3	4	5	6	7	8
$f(x)$	0.01	0.004	0.02	0.12	0.15	0.257	0.325	0.231

(f) Define and express each of the Δ and ∇ in terms of E.

Q.3 Attempt Any Three of the following. (15)

(a) Solve the following simultaneous equations by Gauss - Seidel method. Perform four iterations correct to four decimal places.
$30x_1 - 2x_2 + 3x_3 = 75,\ 2x_1 + 2x_2 + 18x_3 = 30,\ x_1 + 17x_2 - 2x_3 = 48$

(b) Solve the following simultaneous equations by Gauss-Jordan elimination method. Perform four iterations correct to four decimal places.
$8x_1 + 2x_2 - 3x_3 = 4, 2x_1 - 5x_2 - 6x_3 = 8, 7x_1 - 2x_2 + 5x_3 = 15$

(c) Evaluate the following using Simpson's 3/8th rule correct up to four decimal places.
$\int_0^2 \log(1 + x)^{1/2} dx$

(d) For the set of points (0,2), (1,−2), (2,−1), evaluate $\left(\frac{dy}{dx}\right)_{x=2}$.

(e) Evaluate $\int_1^2 e^{-\frac{1}{2}x} dx$ by dividing the range into 4 equal parts using Trapezoidal rule.

(f) Evaluate $\int_{1.0}^{1.8} \frac{e^x+e^{-x}}{2} dx$ using Simpson's one-third rule with $h = 0.2$.

Q.4 Attempt Any Three of the following. (15)

(a) Solve $\frac{dy}{dx} = \frac{y-x}{y+x}$, where $y(0) = 1$ to find $y(0.2)$ using second order Runge-Kutta method correct up to four decimal places. Take $h = 0.2$.

(b) Fit a second-degree parabola to the data given below.

x	2.5	3	3.5	4	4.5	5	5.5
y	4.32	4.83	5.27	5.47	6.26	6.79	7.23

(c) Find the best-fit values of a and b so that $y = a + bx$ fits the data given in the table.

x	71	68	73	69	67	65	66	67
y	69	72	70	70	68	67	68	64

(d) Solve $\frac{dy}{dx} = \log(1+x);\ y(1) = 2$ for $x = 1.2$ and using Euler's modified method, taking $h = 0.2$.

(e) Obtain a regression plane equation $y = a + bx + cz$ by using multiple regression to fit the following data.

x	0	2	2.5	1	4	7
z	0	1	2	3	6	2
y	5	10	9	0	3	27

(f) Using Taylor series method, solve $\frac{dy}{dx} = x^2 + y^2$ with $y(0) = 1$. Find $y(0.1)$

Q.5 **Attempt Any Three of the following.** **(15)**

(a) Two food products A and B are available at the cost of Rs. 30 and Rs. 20 per pack. Food A and B contain 80 and 40 units of proteins and 9 and 5 units of vitamin respectively. How many packs of A and B must be purchased so as to meet the requirement of 600 units of proteins and 72 units of vitamins at minimum cost?

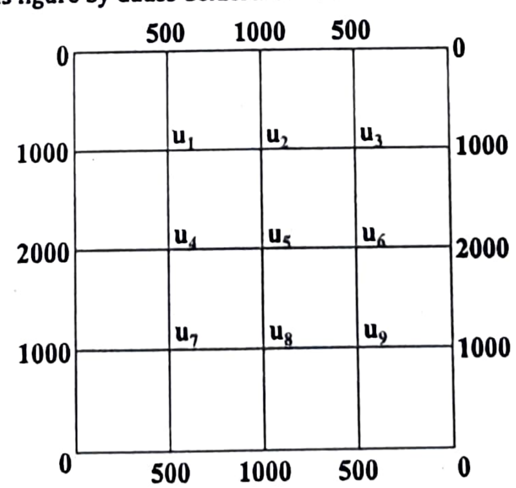
(b) A carpenter has 45, 40 and 25 running feet of teak wood, plywood and rosewood respectively. A table requires 2, 1, 1 running foot and a chair requires 1, 2, 1 running foot of teak wood, plywood and rosewood respectively. If a table is sold for Rs. 4800 and a chair for Rs. 1600, how many tables and chairs should the carpenter make and sell in order to obtain maximum revenue out of his stock of wood? Formulate the above LPP.

(c) Classify the following partial differential equations into elliptic, parabolic and hyperbolic.

(i) $\frac{\partial^2\phi}{\partial x^2} + (x-1)\frac{\partial^2\phi}{\partial y^2} = 0$

(ii) $(1+x^2)\frac{\partial^2 u}{\partial x^2} + (5+2x^2)\frac{\partial^2 u}{\partial x \partial t} + (4+x^2)\frac{\partial^2 u}{\partial t^2} = 0$

(d) Given the values of $u(x,y)$ on the boundary of square mesh with boundary values as shown in the figure. Evaluate the function $u(x,y)$ satisfying the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ at the pivotal points of this figure by Gauss-Seidel method.



(e) Using Crank – Nicholson formula, solve $\frac{\partial^2 u}{\partial x^2} - 16\frac{\partial u}{\partial t} = 0$, given $u(x,0) = 0$, $u(0,t) = 0$, $u(1,t) = 200t$. Compute u for one step t division. Assume $h = 0.25$.

(f) Using Bender – Schmidt method, solve $\frac{\partial^2 u}{\partial x^2} - 32\frac{\partial u}{\partial t} = 0$, given $u(x,0) = 0$, $u(0,t) = 0$, $u(1,t) = t$. Compute u for four step t division. Assume $h = 0.25$.

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